

Quark-gluon vertex model and lattice-QCD data

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A model for the dressed quark-gluon vertex, at zero gluon momentum, is formed from a nonperturbative extension of the two Feynman diagrams that contribute at 1-loop in perturbation theory. The required input is an existing ladder-rainbow model Bethe-Salpeter kernel from an approach based on the Dyson-Schwinger equations; no new parameters are introduced. The model includes an Ansatz for the triple-gluon vertex. Two of the three vertex amplitudes from the model provide a point-wise description of the recent quenched lattice-QCD data. An estimate of the effects of quenching is made.

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I. INTRODUCTION

A great deal of progress in the QCD modeling of hadron physics has been achieved through the use of the ladder-rainbow truncation of the Dyson-Schwinger equations (DSEs). For two recent reviews, see Refs. [1] and [2]. Apart from 1-loop renormalization group improvement, this truncation is built upon a bare quark-gluon vertex. Recent investigations with simple dressed vertex models have indicated that material contributions to a number of observables are possible with a better understanding of the infrared structure of the vertex. These diverse model indications include an enhancement in the quark condensate [3, 4], an increase of about 300 MeV in the b_1/h_1 axial vector meson mass [5], and about 200 MeV of attraction in the ρ/ω vector meson mass.

In the absence of well-constrained nonperturbative models for the vertex, it has often been assumed that a reasonable beginning is the (Abelian) Ball-Chiu Ansatz [6] times the appropriate color matrix. An example is provided by the recent results from a truncation of the gluon-ghost-quark DSEs where this vertex dressing contributes materially to a reasonable quark condensate value [4]. However, there is no known way to develop a Bethe-Salpeter (BSE) kernel that is dynamically matched to a quark self-energy defined in terms of such a phenomenological dressed vertex in the sense that chiral symmetry is preserved through the axial-vector Ward-Takahashi identity. The latter implementation of chiral symmetry guarantees the Goldstone boson nature of the flavor non-singlet pseudoscalars independently of model details [7]. There is a known constructive scheme [8] that defines a diagrammatic expansion of the BSE kernel corresponding to any diagrammatic expansion of the quark self-energy such that the axial-vector Ward-Takahashi identity is preserved. For this reason, recent nonperturbative vertex models have employed simple diagrammatic representations [3, 5, 9].

It is only recently that lattice-QCD has begun to provide information on the infrared structure of the dressed quark-gluon vertex [10]. In this work we generate a model dressed vertex, for zero gluon momentum, based on an

Ansatz for non-perturbative extensions of the only two diagrams that contribute at 1-loop order in perturbation theory. An existing ladder-rainbow model kernel is the only required input. We compare to the recent lattice-QCD data without parameter adjustment.

In Section II we recall the vertex to 1-loop in perturbation theory and point out the structure and properties that are used to suggest the Ansatz for non-perturbative extension. The non-perturbative extension is described in Section III and the results are presented and discussed in Section IV.

II. ONE-LOOP PERTURBATIVE VERTEX

We denote the dressed-quark-gluon vertex for gluon momentum k and quark momentum p by $ig t^c \Gamma_\sigma(p+k, p)$, where $t^c = \lambda^c/2$ and λ^c is an SU(3) color matrix. Through $\mathcal{O}(g^2)$, i.e., to 1-loop, the amplitude Γ_σ is given, in terms of Fig. 1, by^[i]

$$\Gamma_\sigma(p+k, p) = Z_{1F} \gamma_\sigma + \Gamma_\sigma^A(p+k, p) + \Gamma_\sigma^{NA}(p+k, p) + \dots, \quad (1)$$

with

$$\begin{aligned} \Gamma_\sigma^A(p+k, p) = & -(C_F - \frac{C_A}{2}) \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \gamma_\mu \\ & \times S_0(q+k) \gamma_\sigma S_0(q) \gamma_\nu, \end{aligned} \quad (2)$$

and

$$\begin{aligned} \Gamma_\sigma^{NA}(p+k, p) = & -\frac{C_A}{2} \int_q^\Lambda g^2 \gamma_\mu S_0(p-q) \gamma_\nu D_{\mu\mu'}(q+k) \\ & \times i\Gamma_{\mu'\nu'\sigma}^{3g}(q+k, q) D_{\nu'\nu}(q), \end{aligned} \quad (3)$$

where $\int_q^\Lambda = \int^\Lambda d^4q/(2\pi)^4$ denotes a loop integral regularized in a translationally-invariant manner at mass-scale Λ . Here $Z_{1F}(\mu^2, \Lambda^2)$ is the vertex renormalization

[i] We employ Landau gauge and a Euclidean metric, with: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$; $\gamma_\mu^\dagger = \gamma_\mu$; and $a \cdot b = \sum_{i=1}^4 a_i b_i$.

constant to ensure $\Gamma_\sigma = \gamma_\sigma$ at renormalization scale μ . The following quantities are bare: the three-gluon vertex $ig f^{abc} \Gamma_{\mu\nu\sigma}^{3g}(q+k, q)$, the quark propagator $S_0(p)$, and the gluon propagator $D_{\mu\nu}(q) = T_{\mu\nu}(q) D_0(q^2)$, where $T_{\mu\nu}(q)$ is the transverse projector. The next order terms in Eq. (1) are $\mathcal{O}(g^3)$: the contribution involving the four-gluon vertex, and $\mathcal{O}(g^4)$: contributions from crossed-box and two-rung gluon ladder diagrams, and 1-loop dressing of the triple-gluon vertex, etc.

The color factors in Eqs. (2) and (3), given by

$$\begin{aligned} t^a t^b t^a &= (C_F - \frac{C_A}{2}) t^b = -\frac{1}{2N_c} t^b \\ t^a f^{abc} t^b &= \frac{C_A}{2} i t^c = \frac{N_c}{2} i t^c \end{aligned} \quad (4)$$

reveal two important considerations. The color factor of the (Abelian-like) term Γ_σ^A would be given by $t^a t^a = C_F = (N_c^2 - 1)/2N_c$ for the strong dressing of the photon-quark vertex, i.e., in the color singlet channel. The octet Γ_σ^A is of opposite sign and is suppressed by a factor $1/(N_c^2 - 1)$: single gluon exchange between a quark and antiquark has relatively weak repulsion in the color-octet channel, compared to strong attraction in the color-singlet channel. Net attraction for the gluon vertex (at least to this order) is provided by the non-Abelian $\Gamma_\sigma^{\text{NA}}$ term, which involves the three-gluon vertex: the color factor is amplified by $-N_c^2$ over the Γ_σ^A term.

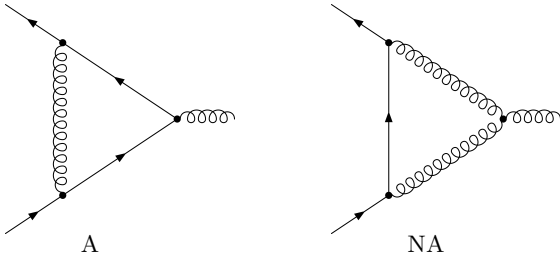


FIG. 1: The quark-gluon vertex at one loop. The left diagram labelled A is the Abelian-like term Γ_σ^A , and the right diagram labelled NA is the non-Abelian term $\Gamma_\sigma^{\text{NA}}$.

The specific form of the bare triple-gluon vertex is conveniently expressed in terms of three momenta $p_1 = q+k$, $p_2 = -q$ and $p_3 = -k$, that are outgoing. Thus with $\Gamma_{\mu\nu\sigma}^{3g}(q+k, q) \equiv \tilde{\Gamma}_{\mu\nu\sigma}^{3g}(p_1, p_2, p_3)$, we have

$$\begin{aligned} \tilde{\Gamma}_{\mu\nu\sigma}^{3g}(p_1, p_2, p_3) &= -\{ (p_1 - p_2)_\sigma \delta_{\mu\nu} + (p_2 - p_3)_\mu \delta_{\nu\sigma} \\ &\quad + (p_3 - p_1)_\nu \delta_{\sigma\mu} \} \end{aligned} \quad (5)$$

and the complete vertex is symmetric under permutations of all gluon coordinates. In Landau gauge $\Gamma_{\mu\nu\sigma}^{3g}$ obeys the Slavnov-Taylor identity

$$k_\sigma \Gamma_{\mu\nu\sigma}^{3g}(q+k, q) = D_0^{-1}(q) T_{\mu\nu}(q) - D_0^{-1}(q+k) T_{\mu\nu}(q+k). \quad (6)$$

The nonperturbative model of Section III addresses the $k=0$ case and makes an extension of the bare result

$$\Gamma_{\mu\nu\sigma}^{3g}(q, q) = -\frac{\partial}{\partial q_\sigma} D_0^{-1}(q) T_{\mu\nu}(q) \quad , \quad (7)$$

which allows the amplitude for the non-Abelian diagram at $k=0$ to take the form

$$\begin{aligned} \Gamma_\sigma^{\text{NA}}(p, p) &= -i \frac{C_A}{2} \int_q^\Lambda \gamma_\mu S_0(p-q) \gamma_\nu \\ &\quad \times \left\{ \frac{\partial}{\partial q_\sigma} g^2 D_0(q^2) \right\} T_{\mu\nu}(q) \quad . \end{aligned} \quad (8)$$

It is easy to verify that the Abelian diagram gives

$$\Gamma_\sigma^A(p, p) = -i \left[1 - \frac{C_A}{2} C_F^{-1} \right] \frac{\partial}{\partial p_\sigma} \Sigma^{(1)}(p) \quad , \quad (9)$$

in terms of the 1-loop self-energy.

The dressing provided by the combination $\Gamma_\sigma^A + \Gamma_\sigma^{\text{NA}}$ yields a vertex that satisfies the Slavnov-Taylor identity (STI) through $\mathcal{O}(g^2)$ [11]. This identity expresses the divergence of the vertex in terms of the bare and 1-loop contributions to three objects: $S(p)^{-1}$, the ghost propagator dressing function, and the ghost-quark scattering amplitude. The 1-loop $S(p)^{-1}$ part of this relation is generated partly from Γ_σ^A (with a weak repulsive color strength) and partly from $\Gamma_\sigma^{\text{NA}}$ (with the complementary strongly attractive color strength). The $\Gamma_\sigma^{\text{NA}}$ term also provides the explicitly non-Abelian terms of the $\mathcal{O}(g^2)$ STI.

III. NONPERTURBATIVE VERTEX MODEL

The general nonperturbative vertex at $k=0$ has a representation in terms of three invariant amplitudes; here we choose

$$\Gamma_\sigma(p, p) = \gamma_\mu \lambda_1(p^2) - 4p_\mu \gamma \cdot p \lambda_2(p^2) - i2p_\mu \lambda_3(p^2) \quad . \quad (10)$$

since the lattice-QCD data [10] is provided in terms of these $\lambda_i(p^2)$ amplitudes. A useful comparison is the corresponding vertex in an Abelian theory like QED; it is given by the Ward identity $\Gamma_\sigma^{WI}(p, p) = -i \partial S^{-1}(p) / \partial p_\sigma$ in terms of the exact propagator $S^{-1}(p)$. With $S^{-1}(p) = i\gamma \cdot p A(p^2) + B(p^2)$, this leads to the correspondence $\lambda_1^{\text{WI}} = A$, $\lambda_2^{\text{WI}} = -A'/2$, and $\lambda_3^{\text{WI}} = B'$, where $f' = \partial f(p^2) / \partial p^2$.

Our nonperturbative model for the dressed quark-gluon vertex is defined by extensions of Eqs. (2) and (3) into dressed versions determined solely from an existing ladder-rainbow model DSE kernel that has 1-loop QCD renormalization group improvement. Two DSE models are employed. The first model (DSE-Lat) [12] represents a mapping of quenched lattice data for the gluon propagator [13] into a continuum ladder-rainbow model kernel having sufficient effective infrared vertex strength to

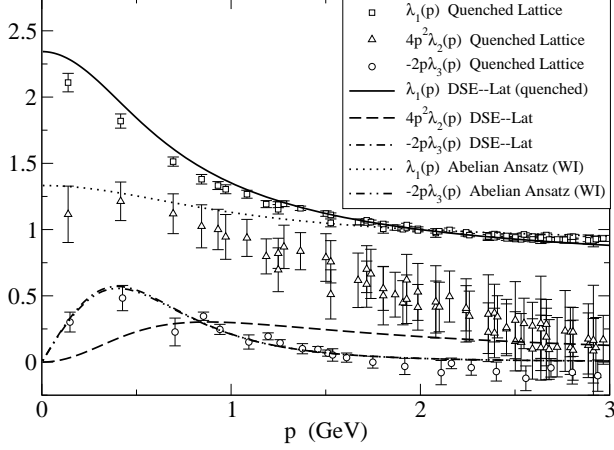


FIG. 2: The amplitudes of the dressed quark-gluon vertex at zero gluon momentum and for quark current mass $m(\mu = 2 \text{ GeV}) = 60 \text{ MeV}$. Quenched lattice data [10] is compared to the results of the DSE-Lat model [12]. The Abelian Ansatz (Ward identity) is also shown except for $\lambda_2(p)$ which is almost identical to the DSE-Lat model.

reproduce quenched lattice data for the quark propagator [14]. In this sense, it represents quenched dynamics. The second (DSE-MT) [15] provides a good one-parameter fit to a wide variety of light quark meson physics; in this sense it represents unquenched dynamics. Both can be implemented through the substitution $g^2 D_0(q^2) \rightarrow \mathcal{G}(q^2)/q^2$ in the ladder BSE kernel that appears in the integrand for Γ_σ^A , in Eq. (2). The bare quark propagators in Eqs. (2) and (3) are replaced by solutions of the quark DSE in rainbow truncation using the appropriate kernel, namely,

$$S(p)^{-1} = Z_2 i \not{p} + Z_4 m(\mu) + C_F \int_{p'}^\Lambda \frac{\mathcal{G}(q^2)}{q^2} T_{\mu\nu}(q) \gamma_\mu S(p') \gamma_\nu, \quad (11)$$

where $q = p - p'$. For both models the effective coupling $\mathcal{G}(q^2)$ has a parameterized form in the infrared, while in the ultraviolet, the QCD factors from 1-loop renormalization of the quark and gluon propagators and the pair of quark-gluon vertices have been absorbed so that $\mathcal{G}(q^2)$ matches $4\pi\alpha_s^{1\text{-loop}}(q^2)$ [16]. The corresponding rainbow DSE solution reproduces the leading logarithmic behavior of the quark mass function in the perturbative space-like region.

A ladder-rainbow model kernel is generally not sufficient to specify a dressed extension of $\Gamma_\sigma^{\text{NA}}(p+k, p)$ from Eq. (3). However at $k = 0$, the expression in Eq. (8) for $\Gamma_\sigma^{\text{NA}}(p, p)$ has combined the triple gluon vertex and the gluon propagators to produce a form that emphasizes the close connection to the ladder kernel and the self-energy integral. The same nonperturbative extension $g^2 D_0(q^2) \rightarrow \mathcal{G}(q^2)/q^2$ as used earlier now suggests itself, and we use it. Our justification for this choice is

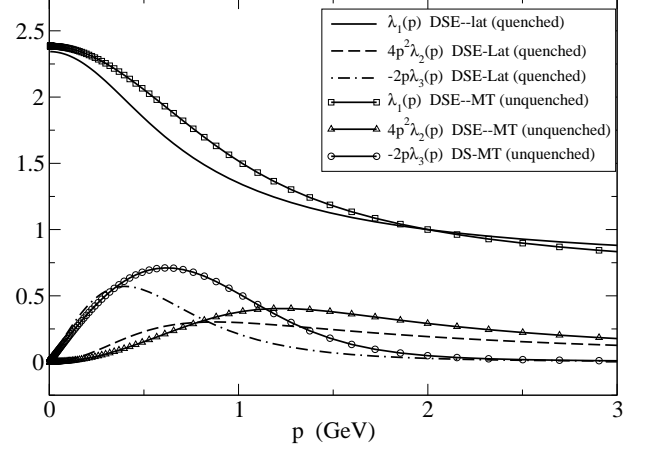


FIG. 3: The amplitudes of the dressed quark-gluon vertex at zero gluon momentum, and for quark current mass $m(\mu = 2 \text{ GeV}) = 60 \text{ MeV}$, from two models: DSE-Lat [12] and DSE-MT [15] that relate to quenched and unquenched content respectively.

one of simplicity; no new parameters are introduced.

IV. RESULTS AND DISCUSSION

In Fig. 2 we display the DSE-Lat model results in a dimensionless form for comparison with the (quenched) lattice data^[ii]. The renormalization scale of the lattice data is $\mu = 2 \text{ GeV}$ where $\lambda_1(\mu) = 1$, $A(\mu) = 1$. We compare to the lattice data set for which $m(\mu) = 60 \text{ MeV}$. The same renormalization scale and conditions have been implemented for both DSE models^[iii]. For λ_1 and λ_3 we also compare with the Abelian Ansatz in which the amplitudes are obtained from the quark propagator through the Ward Identity, which is equivalent to the $k = 0$ limit of either the Ball-Chiu [6] or Curtis-Pennington [19] Ansatz. Without parameter adjustment, the model reproduces the lattice data for λ_1 and λ_3 quite well over the whole momentum range for which data is available. The Abelian Ansatz, while clearly inadequate for λ_1 below 1.5 GeV, reproduces λ_3 . The present lattice data for λ_2 has large errors; it suggests infrared strength that is seriously underestimated by the model. (The Abelian Ansatz for λ_2 is very close to the DSE model and for

[ii] We note that in Ref. [10] both the lattice data, and the Abelian (Ward identity) Ansatz, for $\lambda_3(p)$ are presented as positive. These two sign errors have been acknowledged [17].

[iii] To facilitate change of the scale μ , we have slightly modified both DSE kernels (both originally defined at fixed scale $\mu_0 = 19 \text{ GeV}$) by including the additional kernel strength factor $Z_2^2(\mu^2, \Lambda^2)/Z_2^2(\mu_0^2, \Lambda^2)$ recommended by Maris [18]. This does not alter results for observables.

reasons of clarity, is not displayed.)

The relative contributions to the vertex dressing made by $\Gamma_\sigma^{\text{NA}}$ and Γ_σ^{A} are indicated by the following amplitude ratios at $p = 0$: $\lambda_1^{\text{NA}}/\lambda_1^{\text{A}} = -60$, $\lambda_2^{\text{NA}}/\lambda_2^{\text{A}} = -14$, and $\lambda_3^{\text{NA}}/\lambda_3^{\text{A}} = -12$. Thus the non-Abelian term $\Gamma_\sigma^{\text{NA}}$ dominates to a greater extent than what the ratio of color factors (-9) would suggest; it also distributes its infrared strength to favor λ_1 more so than does Γ_σ^{A} . Since the momentum-dependent shapes of the $\lambda_i^{\text{NA}}(p)$ and $\lambda_i^{\text{A}}(p)$ are quite similar, the present model results could be summarized quite effectively by ignoring Γ_σ^{A} and scaling $\Gamma_\sigma^{\text{NA}}$ up by about 10%.

Due to the definition of the two DSE models, their comparison in Fig. 3 provides an estimate of the effects of the quenched approximation. The effects are moderate within the present DSE model framework. Fig. 3 also suggests that a model including the four gluon vertex as well as the two diagrams of Fig. 1 should be considered, especially for amplitude λ_2 . The question of the importance of the iterations of the diagrams of Fig. 1 also arises. We have estimated such effects by iteration to all orders based on the ladder-rainbow kernel. This amounts to solution of a ladder Bethe-Salpeter integral equation in which the inhomogeneity is our dressed extension of $Z_{1\text{F}}\gamma_\sigma + \Gamma_\sigma^{\text{NA}}(p, p)$ and the kernel term is the dressed extension of $\Gamma_\sigma^{\text{A}}(p, p)$ with the internal γ_σ replaced by $\Gamma_\sigma(q, q)$. This generates very little change—significantly less than the quenching effect evident in Fig. 3. This is due to the small color factor of the kernel term. We have not explored the consequences of using the dressed vertex self-consistently for the internal quark-gluon vertices

of $\Gamma_\sigma^{\text{NA}}$ in Fig. 1-NA.

The nonperturbative Ansatz we have applied to Eq. (8) is equivalent to the use of an effective dressed triple-gluon vertex $\Gamma_{\mu\nu\sigma}^{3g}$ satisfying Eqs. (6) and (7) with the substitution $D_0(q^2) \rightarrow [\mathcal{G}(q^2)/g^2(\mu^2)]/q^2$. Some perturbative studies of $\Gamma_{\mu\nu\sigma}^{3g}$ have been made at 1-loop [20, 21] but they provide no guidance for extension to infrared scales. The nonperturbative Ansätze for $\Gamma_{\mu\nu\sigma}^{3g}$ suggested in Refs. [2] and [22] for use within truncated gluon-ghost-quark DSEs require explicit models for the ghost dressing function and the ghost-gluon vertex that appear in the STI for $\Gamma_{\mu\nu\sigma}^{3g}$. Such considerations are beyond the scope of the present work; they would entail additional parameters that are not warranted at this stage. Recent work on a model of the quark-gluon vertex using such an Ansatz for $\Gamma_{\mu\nu\sigma}^{3g}$ has produced results similar to the present work, except that the $m(\mu) = 115$ MeV case is considered [9]. Evidently the detailed infrared structure of $\Gamma_{\mu\nu\sigma}^{3g}$ is not crucial to present considerations.

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